COMPARISON OF NUMERICAL AND EFFECTIVE-INDEX METHODS FOR A CLASS OF DIELECTRIC WAVEGUIDES*

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Abstract

A numerical method and the effective-index method are applied to a three-layer, constant thickness dielectric waveguide with smoothly varying dielectric constant inside the active layer and constant permittivity in the confining layers. The results of the two methods are compared in terms of the propagation constant γ calculated by each method. Application of the effective-index method facilitates a physical understanding of dielectric waveguide modes as well as providing an efficient approximate method for calculating mode behavior.

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Introduction

Several papers [1-3] have analyzed mode propagation in dielectric waveguides with spatially varying refractive index, usually approximated by parabolic or $\tanh^2$ functions, which go to infinity at large distances from the reference point $x = 0$. We will use the approximation [4]

$$\kappa = \kappa_S + \Delta\kappa/\cosh^2 (x/x_0) \tag{1}$$

to describe the variation of κ. This is in closer correspondence with the physical situation and leads to equations with known solutions. The disadvantage in this case is the fact that the field solutions consist of a finite (possibly empty) set of confined (trapped) modes, an infinite set of diverging "leaky" modes and a continuum of solutions that will be designated as "radiation" modes.

The 2-dimensional character of the problem due to the variation of the refractive index in the lateral (x) and transverse (y) directions requires the use of approximate methods or numerical solutions. Among the former, the effective index method is the most popular ([2], [4] - [8]). Direct numerical integration is possible, but the computation time required is much larger than that needed for other methods like the one used in [1], which we will apply in this paper.

The following sections describe the class of waveguides considered, the numerical method and the application of the effective-index method to our problem, ending with conclusions.

Description of Structures

Figure 1 shows the structure considered in this paper. The confining layers A and C are assumed identical, their refractive indices being described by

$$\kappa_A = \kappa_C \cong n_A^2 - i\,\alpha_A n_A/k_0 \tag{2}$$

α_A describes the power loss in these layers and is constant with distance. The active layer has a constant thickness d, with a refractive index

$$\kappa(x) = n^2(x) - i\,\alpha(x)\,n(x)/k_0 \tag{3}$$

whose dependence with x is considered to be reasonably well-approximated by (1). In that equation,

$$\Delta\kappa = \kappa_0 - \kappa_S \tag{4a}$$

$$\kappa_0 = \kappa(0) = n_0^2 + i\,g_0\,n_0/k_0 \tag{4b}$$

$$\kappa_S = n_S^2 - i\,\alpha_S\,n_S/k_0 \tag{4c}$$

x_0 is a parameter related to the width of the stripe contact in the case of a semiconductor laser. The values of power attenuation coefficient and refractive index inside the active region far away from the stripe are α_S and n_S, respectively. The quantity g_0 represents the peak power gain under the stripe $(x = 0)$, where the refractive index is n_0. For $\Delta n = n_0 - n_S > 0$ the mode will be index-guided, while for $\Delta n < 0$ it will be index anti-guided, and this latter effect can eventually offset the guiding effect of the gain distribution.

Numerical Solution

We follow the method used in [1]. Maxwell's equations are solved for both the active and the confining layers, requiring as boundary conditions that the general solutions inside and outside the active layer and their normal derivatives match at the interfaces $y = \pm d/2$. We also demand that these solutions vanish at $x = \pm\infty$, $y = \pm\infty$. Application of the method of separation of variables results in a vertical field solution inside the active region of the form

$$\phi(y) = \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} qy \tag{5}$$

and a differential equation

$$\frac{d^2\psi}{dx^2} + \left[k_0^2\,\kappa_S + \gamma^2 - q^2 + k_0^2\,\frac{\Delta\kappa}{\cosh^2(x/x_0)}\right]\psi = 0 \tag{6}$$

where q is the separation constant. Equation (6) possesses solutions of the form

$$\psi_\ell(x) = [\cosh(x/x_0)]^{\ell - b_0}\, C_\ell^{b_0 - \ell + \frac{1}{2}}(\tanh[x/x_0]) \tag{7}$$

where $C_\ell^\lambda(z)$ are Gegenbauer polynomials [9] and

$$b_0 = -\tfrac{1}{2} + (\tfrac{1}{4} + k_0^2\,x_0^2\,\Delta\kappa)^{\frac{1}{2}} \tag{8}$$

To satisfy the boundary conditions we require

$$\mathrm{Re}\{b_0 - \ell\} > 0 \tag{9}$$

For $\mathrm{Re}\{b_0\} > 0$, the fundamental mode $(\ell=0)$ is trapped. Modes of order ℓ such that $\mathrm{Re}\{b_0 - \ell\} < 0$ are still solutions of (6) but diverge as $|x| \to \infty$ and will be designated as "leaky". Radiation modes would be described by other solutions of (6) for arbitrary (non-integer) eigenvalues [9]. The general solution will be a linear combination of the few discrete trapped modes (8) satisfying (9), plus an integral over the continuum, and the leaky modes must be excluded if the field has to vanish for $|x| \to \infty$. The importance of the continuum can be expected to decrease as the number of trapped modes increases. Since this number is relatively large for structures with modal gains that are high and not very sensitive to the refractive index step $\Delta n = n_0 - n_S$, this continuum will be neglected. Hence, the general even solution inside the active region is approximated by

$$\Psi_b(x,y) \cong \sum_{\ell=0,2,\ldots}^{\bar{\ell}_{max}} A_\ell \, \psi_\ell(x) \cos(q_\ell \, y) \tag{10}$$

where $\bar{\ell}_{max}$ is the maximum even value of ℓ for which $\psi_\ell(x)$ is confined. Solution of the wave equation for the confining layers proceeds as in [1]. Matching the functions and their derivatives with respect to y at the boundary $y = d/2$ and applying the orthogonality relation for the trapped modes results in a finite system of homogeneous linear equations

$$(\Omega^T - I)\bar{A} = 0 \tag{11}$$

where $\bar{A}^T = (A_0, A_2, \ldots, A_{\bar{\ell}_{max}})$, I is the unit matrix, and the matrix elements of Ω are given by

$$\Omega_{\ell\ell'}(\gamma) = \frac{4\cos(q_\ell \, d/2)\sqrt{N_\ell}}{\pi x_0 \, q_{\ell'} \sin(q_{\ell'} \frac{d}{2}) \sqrt{N_{\ell'}}} \; I_{\ell\ell'}(\gamma) \tag{12}$$

$$\text{where} \quad I_{\ell,\ell'}(\gamma) = \int_0^\infty \bar{\psi}_\ell(\chi) \, \bar{\psi}_{\ell'}(\chi) \, (\chi^2-\gamma^2-k_a^2)^{\frac{1}{2}} \, d\chi \tag{13}$$

$\bar{\psi}_\ell(\chi)$ being the normalized Fourier cosine transform of $\psi_\ell(x)$, and $\ell, \ell' = 0, 2, \ldots, \bar{\ell}_{max}$. The q_ℓ satisfy

$$q_\ell^2 = \gamma^2 + k_0^2 \, \kappa_S + (b_0 - \ell)^2 / x_0^2 \tag{14}$$

The system of equations (11) has a non-trivial solution only if $\det(\Omega - I) = 0$. Numerical computation of the roots yields the possible values of the propagation constant γ.

Numerical Results

The method was applied to a structure described by the following parameters:

$n_A = 3.38$, $\alpha_A = 50\ cm^{-1}$, $\alpha_S = 50\ cm^{-1}$, $g_0 = 200 cm^{-1}$

$n_0 = 3.595$, $x_0 = 6\mu m$.

Direct solution of (11) involves computation of the $\Omega_{\ell\ell'}$. In general, $\Omega_{\ell\ell'} \neq \Omega_{\ell'\ell}$, but $I_{\ell\ell'} = I_{\ell'\ell}$. Values of the modal loss = $Re\{\gamma\}$ were computed by evaluating $I_{\ell\ell'}(\gamma)$ using the exact expression (13). This has to be done once for every value of γ. Instead of solving the complete system (11), we start with a 1x1 matrix, go on to a 2x2 matrix, etc., and observe that the result converges relatively fast for a 2x2 case, which is considered sufficient for our approximation. In spite of this, the computation time is still impractically high. An increase in speed by nearly two orders of magnitude while still maintaining good accuracy is achieved expanding the radical in (13) using the binomial theorem (which results in integrals that do not depend on γ) and retaining the first three terms.

Figure 2 shows plots of the modal loss = $Re\{\gamma\}$ as a function of Δn with g_0, the gain under the stripe, as a parameter. We notice that, for each value of g_0, Δn can be decreased up to a certain value beyond which the fundamental mode becomes leaky.

Effective-Index Calculation

The effective-index method consists basically of reducing a two-dimensional problem to an equivalent one-dimensional one. In our case, the two-dimensional character of the problem is given by the dependence of the dielectric constant on x and y. As a first approximation, the variation in one direction (in our case, x) is neglected; this is justified if this variation is much less than that in the y direction. This is equivalent to approximating the waveguide with a simple 3-layer guide whose dielectric constants do not vary with x. The solution of this problem yields the transverse or vertical variation of the field. Next, the original equation describing the 2-dimensional equation in x can be solved for the lateral variation of the field, and the overall solution is approximated by the product of this lateral field and the vertical field found from the 3-layer problem. We start with the wave equation in 2 dimensions:

$$\nabla_t^2 \Psi + [\gamma^2 + k_0^2 \, \kappa(x,y)] \, \Psi = 0 \tag{15}$$

For the simple 3-layer guide we assume $\kappa(x) = \kappa_0$ inside the active layer. Now, we transform (15) making $\Psi(x,y) = \psi(x)\,\phi(y)$, multiplying it by $\phi^*(y)$ and integrating it over y from $-\infty$ to ∞, and obtain

$$\frac{d^2\psi}{dx^2} + [\gamma^2 - q_{eff}^2 + k_0^2 \, \kappa_{S\ eff} + k_0^2 \, \frac{\Delta\kappa_{eff}}{\cosh^2(x/x_0)}]\psi = 0 \tag{16}$$

$$\text{with} \quad q_{eff}^2 = \Gamma \, k_0^2 \, x_0 - p^2 - k_0^2 \, \kappa_A \tag{17a}$$

$$\kappa_{S\ eff} = \Gamma \, \kappa_S \tag{17b}$$

$$\Delta\kappa_{eff} = \Gamma \, \Delta\kappa \tag{17c}$$

where Γ is the filling or confinement factor. Equation (16) has the same form as eq. (6). It will also have polynomial solutions similar to (7) that represent trapped modes:

$$\psi_\ell(x) = [\cosh(x/x_0)]^{\ell - b_{0}eff} \; C_\ell^{b_0 eff - \ell + \frac{1}{2}} (\tanh \frac{x}{x_0}) \tag{18}$$

$$\ell = 0, 1, 2, \ldots$$

where $b_{0\ eff}$ is defined as in (8) with $\Delta\kappa$ replaced by $\Gamma\ \Delta\kappa$ and p is the eigenvalue of the simple 3-layer problem For the fundamental mode, $\ell = 0$, and we obtain

$$\gamma^2 = -k_0^2 \, \kappa_A - (b_{0\ eff}^2 / x_0^2) - (p^2 - k_0^2 \, \Gamma \, \Delta\kappa) \tag{19}$$

for the propagation constant.

Discussion

Values of γ obtained using (19) are compared with those obtained with the numerical method in fig. 3. Solutions are very close for all values of Δn for which the mode exhibits a gain which is relatively high and with low sensitivity to Δn. The results differ most in the range of Δn for which the mode has a net loss or has a relatively low gain with higher sensitivity to Δn.

Figure 4 gives the required value of the peak power gain g_0 under the stripe to obtain a given modal gain G, as a function of Δn, using the effective-index method.

Also included is the region for which the fundamental mode becomes leaky. The vertical confinement factor Γ did not vary appreciably with Δn; a typical value for the case considered was Γ ≈ 0.4963. We see that lateral variations in the refractive index affect the gain much more by altering the lateral field distribution than by affecting the vertical variation.

The effective-index method is seen to be a fast and relatively accurate way to obtain the field distributions for the class of waveguides considered, for which the numerical method we used is not practical for extensive modeling due to its long computation time in spite of all approximations. The remarkable agreement between the effective-index method and experimental results found by other workers ([10]) increases our confidence in this powerful approximate method.

<u>References</u>

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<u>Figures</u>

1. Waveguide structure considered
2. Modal loss as a function of Δn with peak power gain g_0 as a parameter. Regions at left of vertical lines correspond to leaky fundamental mode.
3. Modal loss Re{γ} as a function of Δn for numeric (1x1 matrix) and effective-index methods.
4. Peak power gain under contact stripe (g_0) as a function of Δn with modal gain as a parameter. Shaded area shows region corresponding to leaky fundamental mode.

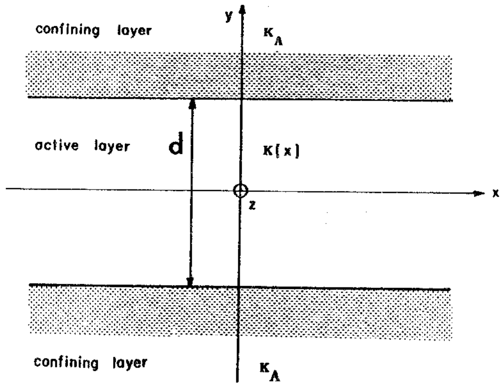

Figure 1

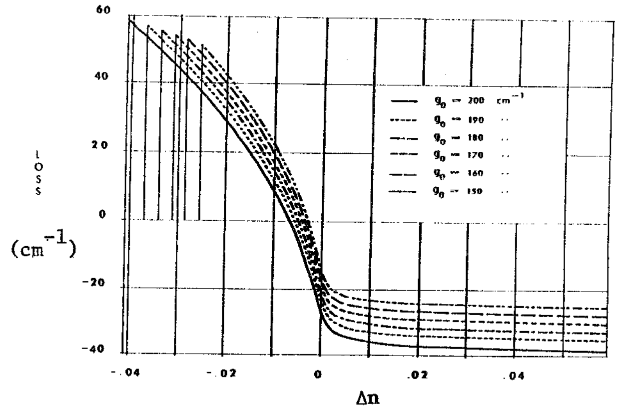

Figure 2

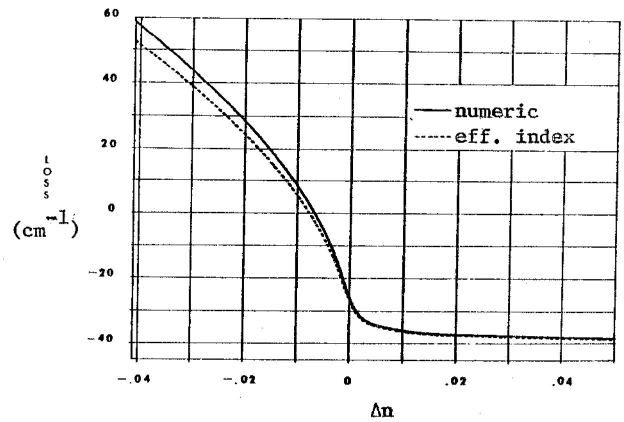

Figure 3

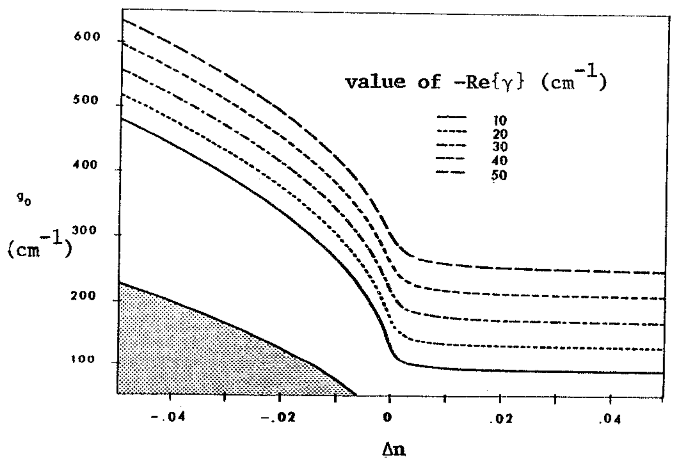

Figure 4